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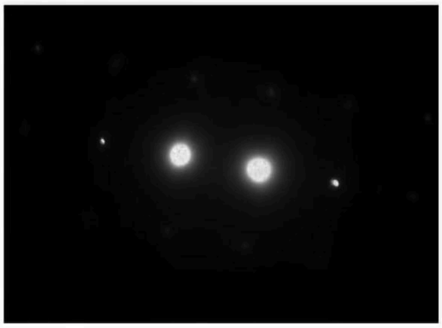
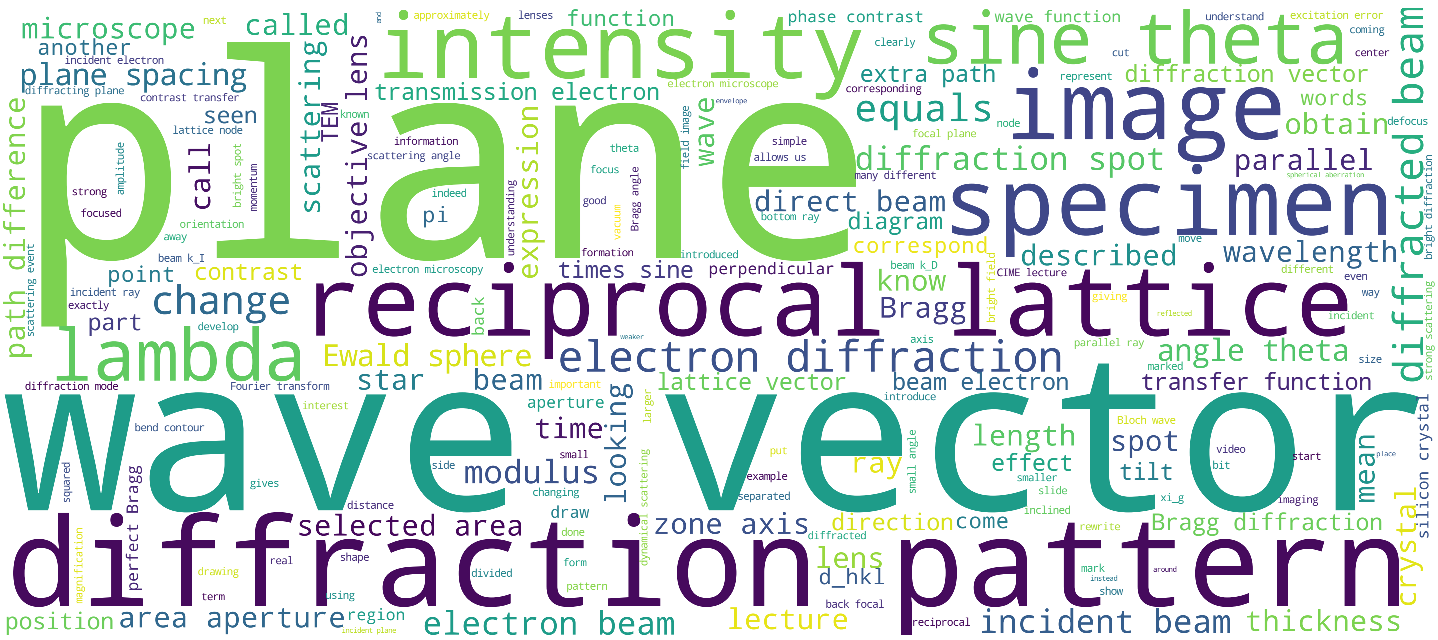


# Basics of electron diffraction: 2-beam diffraction

## Transmission Electron Microscopy

Prof. C. Hébert & Dr D. Alexander

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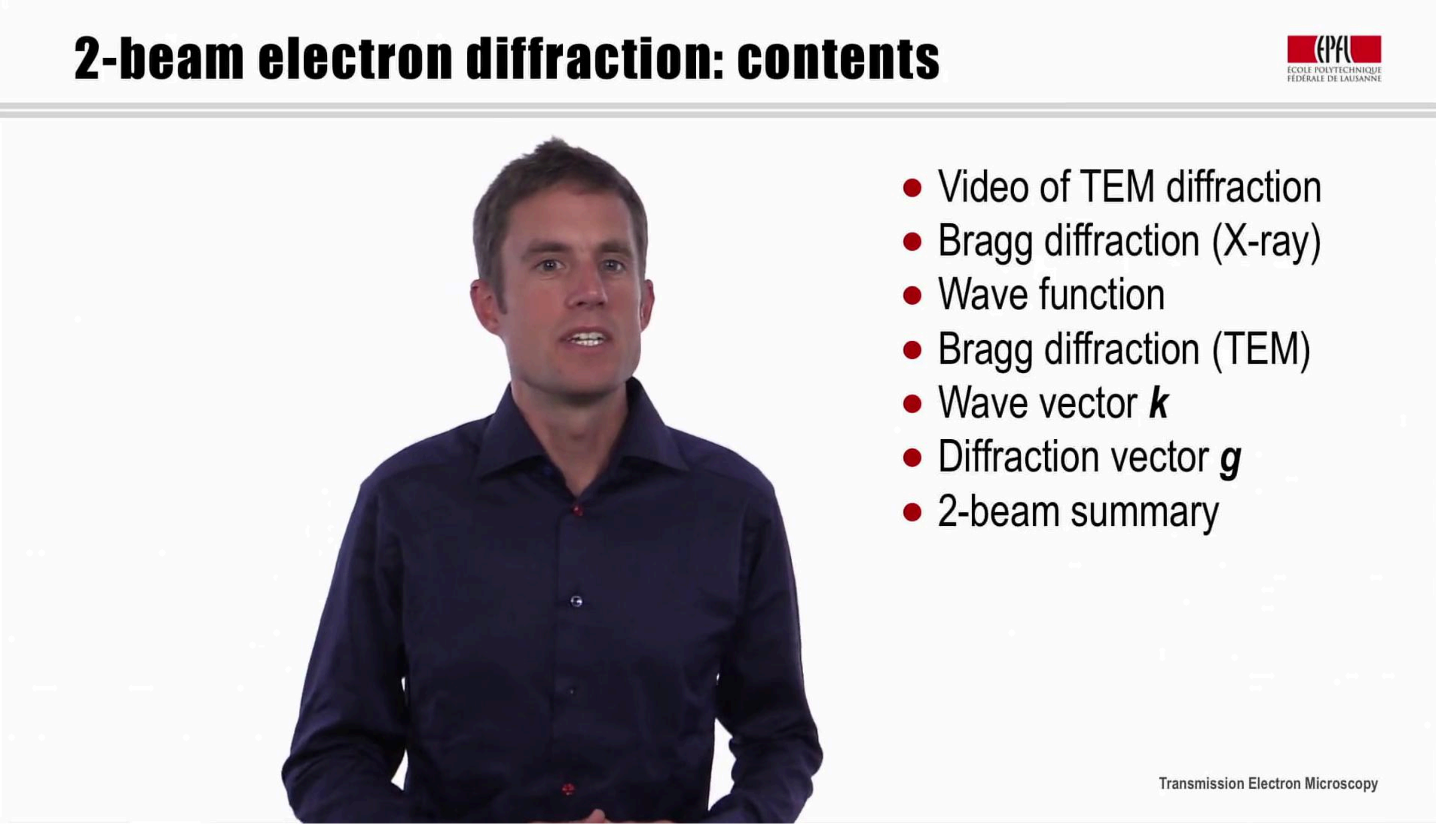
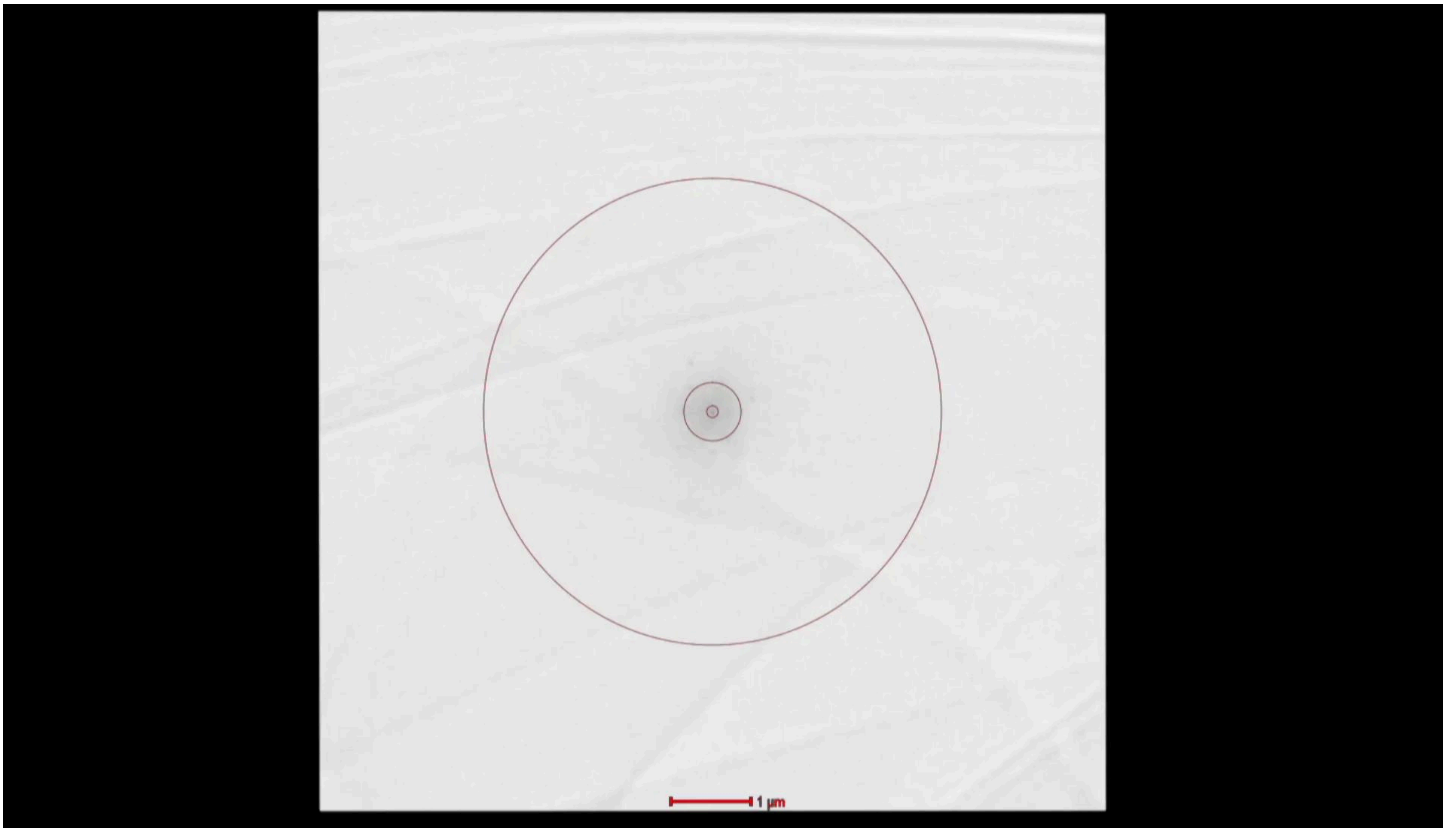



Welcome to CIME's lectures on transmission electron microscopy for materials science. The coming lecturers are dedicated to the basics of electron diffraction. In them, we will develop a generally applicable framework for understanding and interpreting electron diffraction, through the introduction of the Ewald sphere and the reciprocal lattice. While essentially these form a geometrical construct, rather than giving a precise description of the underlying physics of diffraction, we shall see that this construct still provides many insights into the phenomena of electron diffraction in a simple and intuitive way. Before we begin on this construct, in this first lecture on electron diffraction, we will concentrate on the two-beam electron diffraction condition.

Notes

Summary

0m 05s

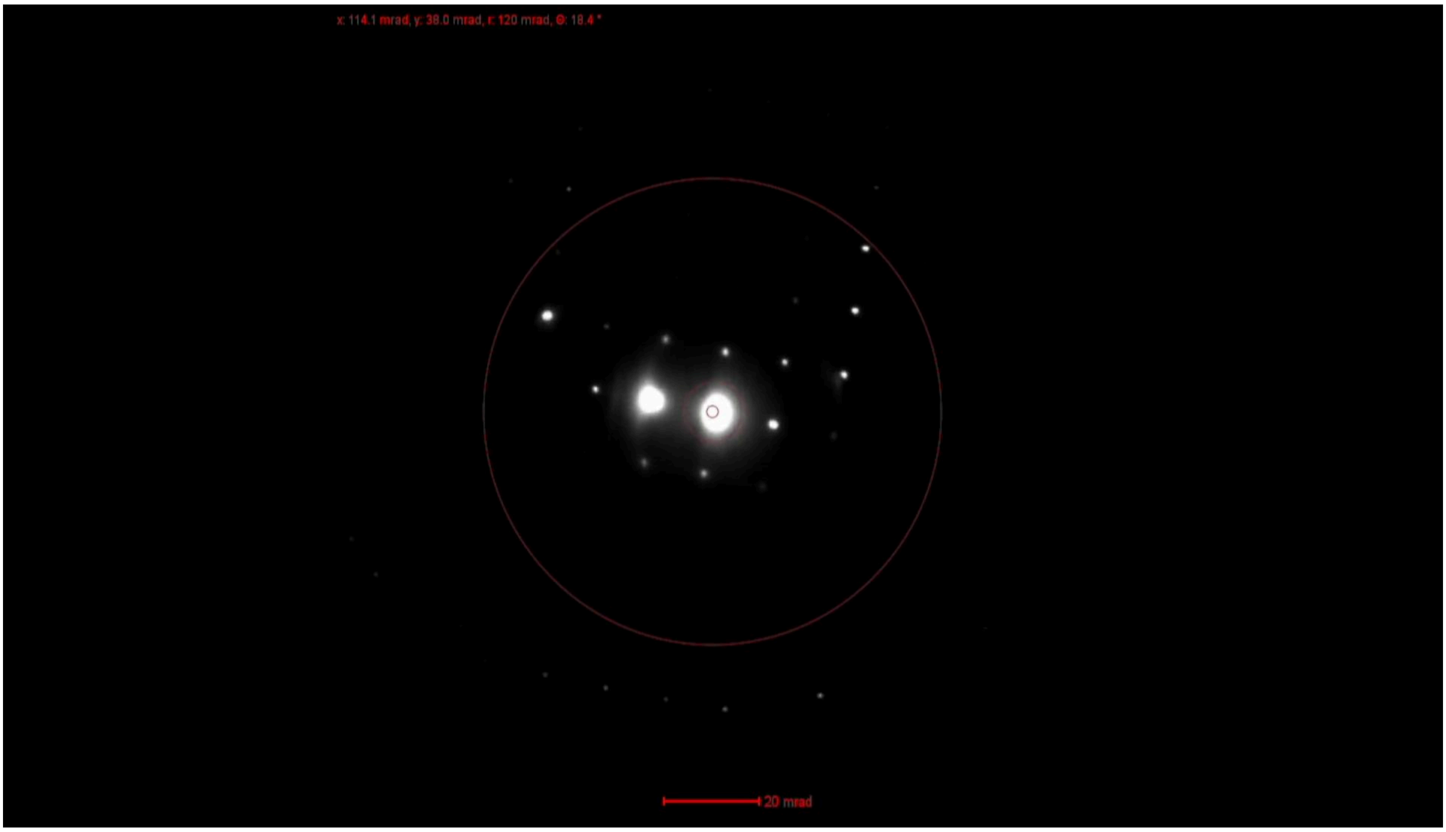



In this video, recorded at the microscope, we will look at electron diffraction from a silicon crystal. At the beginning of the video, now, we are just looking at vacuum. And we are going to introduce a selected area aperture and take the diffraction pattern from that vacuum. Here is the selected area aperture. As explained in the earlier lectures by Cécile Hébert, this aperture is in the image plane, and we use it to select the region of interest of our sample from which we will make a diffraction pattern. In this case, from the vacuum. Now we go to diffraction mode. This is done by changing the strength of the intermediate lens system such that the back focal plane is projected onto our screen. And we see that we have a single bright spot. This is because we have an incident beam made of parallel rays which, in the back focal plane, are being focused to one spot by the objective lens. We can call this spot the direct beam. Now I'm going to leave diffraction mode, go back to image mode, take out that selected area aperture, and then just move the sample until we are looking at some silicon crystal. Here we are moving across the silicon crystal, and choosing a region of interest.

Notes

Summary

0m 51s

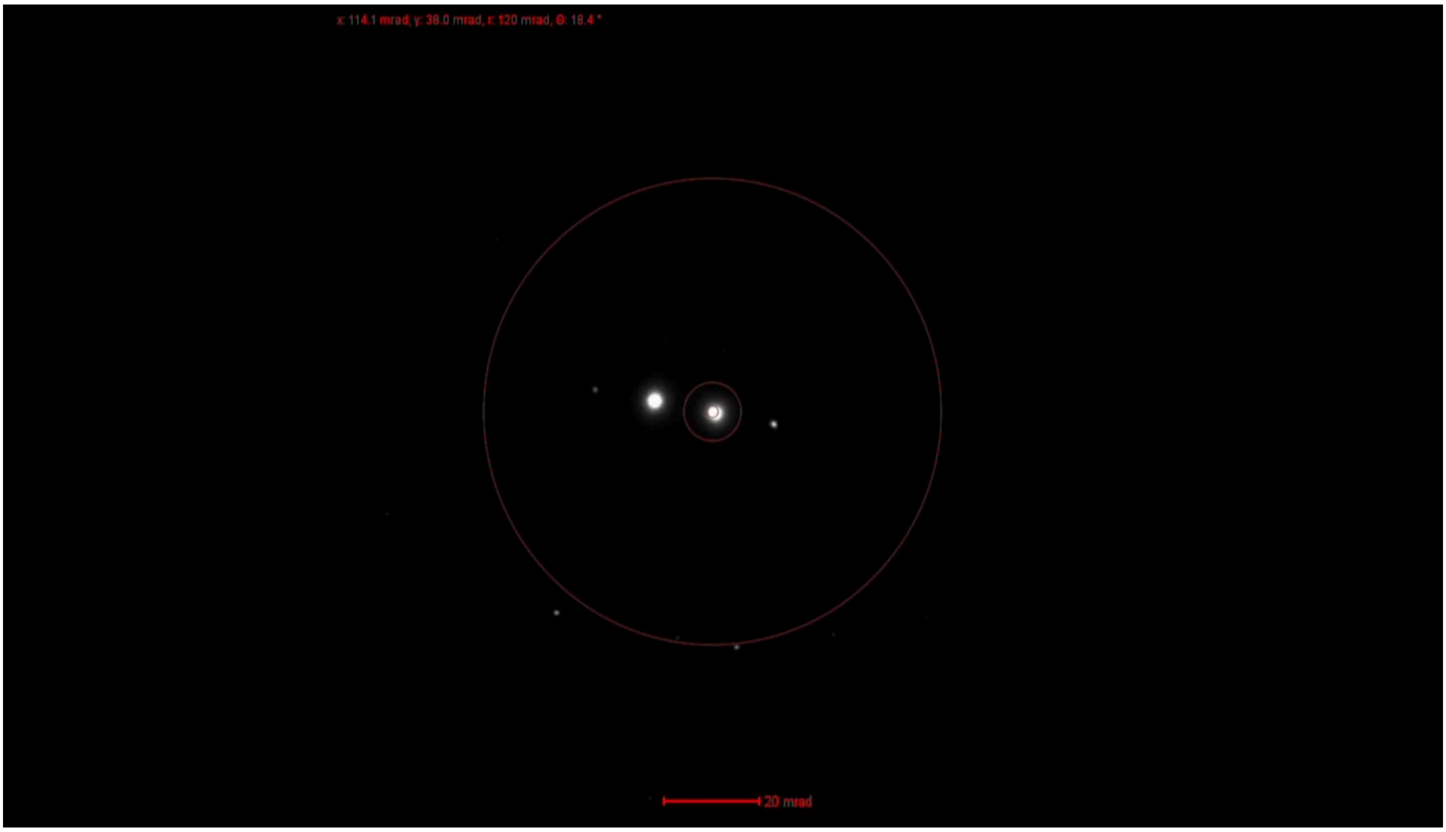



We are going to re-insert the selected area aperture, as so, and once again go to diffraction mode. At first we see various bright spots, each associated with a particular diffracting plane. However, we are not at a very specific orientation of the crystal sample with respect to the electron beam. But with the microscope's goniometer, we will now tilt that sample towards a specific orientation. One where the silicon crystal will be aligned such that its [0 0 1] lattice vector is parallel to the electron beam. As the sample is tilting, we see that different diffraction spots, which are sometimes called reflections, become brighter or weaker in intensity. Their intensity varies depending on the sample orientation. Now we have become aligned on that [0 0 1] orientation, what is referred to as the zone axis of the diffraction pattern, where we now see a lot of intensity in many different diffraction spots and a high symmetry in their pattern. Now we are tilting away from that zone axis condition to try and find another diffraction condition, what we will call the two-beam electron diffraction condition. So, we are already starting to get there.

Notes

Summary

2m 16s

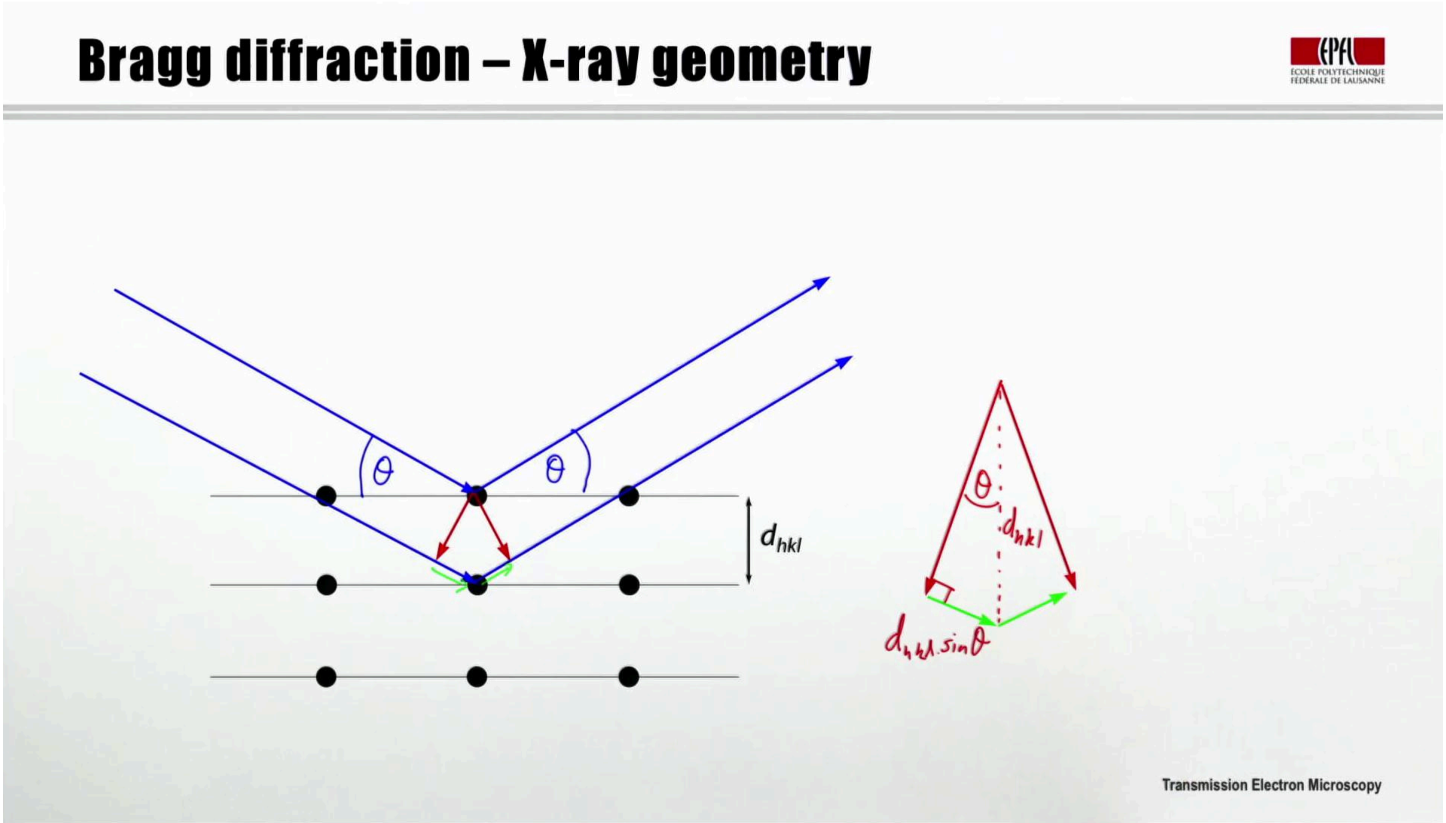



We can see that two of the diffraction spots are becoming brighter and brighter in intensity and the rest weaker and weaker. It takes some refining to get there. So a bit more sample tilt until we will obtain a good two-beam condition. Almost there. And here we are at that two-beam condition. By adjusting the contrast and brightness of the display of the diffraction pattern, we see that, in this two-beam condition, we now have two very bright diffraction spots. All the other diffraction spots have become very weak in intensity compared to these two.

Notes

Summary

3m 33s

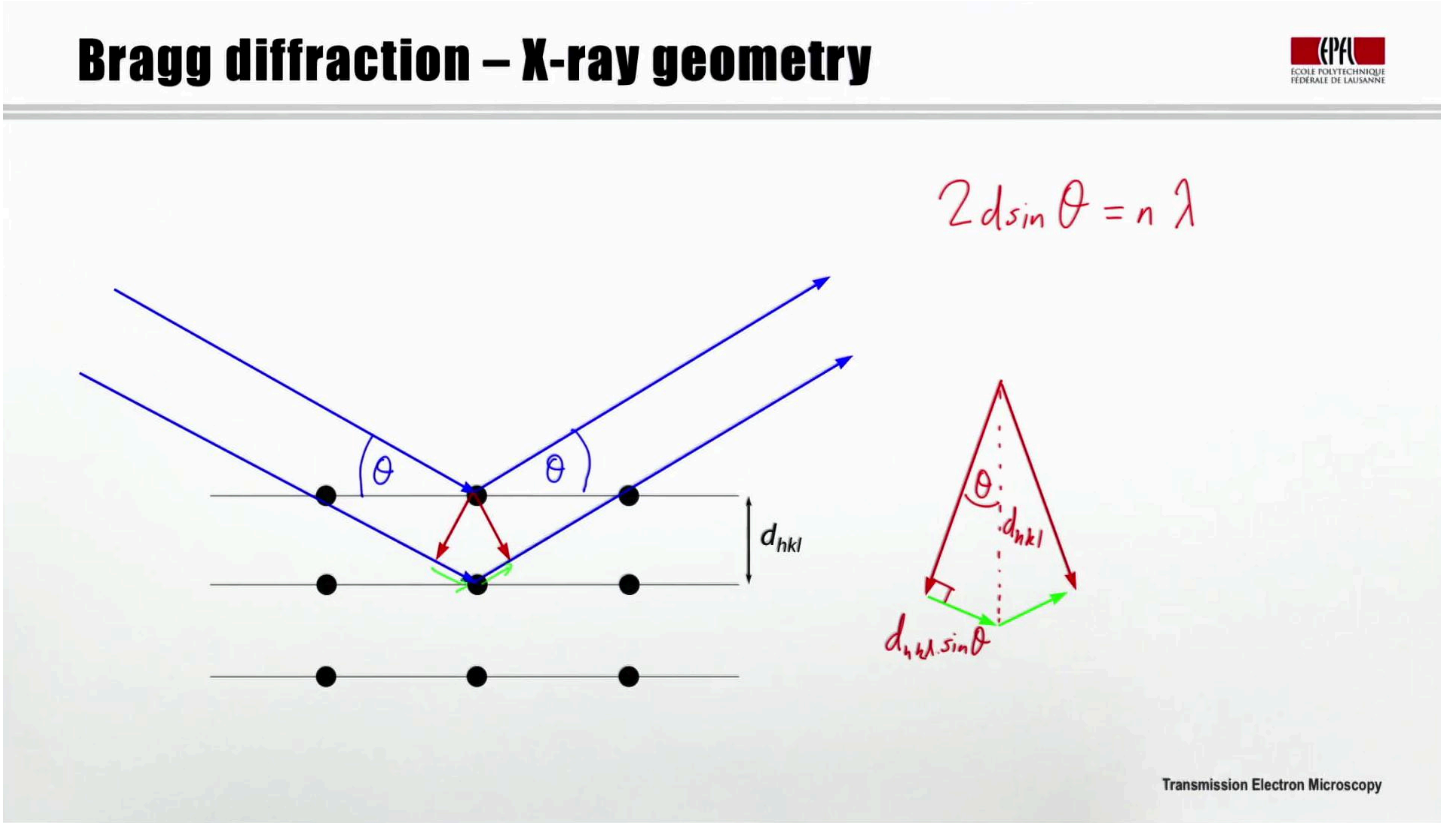



To look at this case of two-beam electron diffraction, we will look once again at the Bragg diffraction geometry. Here we have a crystal with planes, and the planes are separated by a spacing d_hkl, where (h k l) are the Miller indices of the plane. And we are going to consider an incident ray coming into the first plane at an angle theta. And then that ray will be reflected from that plane at the same angle theta. Now we are going to consider another parallel ray reflecting from the plane below. So again, coming in at angle theta, and being reflected by this angle theta. Now because we have an incident plane wave, we know that the waves on these two rays will arrive in phase with each other. However, we can also see that the bottom ray travels a further path difference. So you mark in an orthogonal here, and an orthogonal here. We can see that there is an extra path difference of the bottom ray compared to the top ray. Now if we magnify in on this region here, as so, and look at that extra path difference, we can see that, by trigonometry, here we have an angle theta. This distance here is d_hkl. This is a perpendicular, hence this little bit of path difference here is equal to d_hkl times sine theta.

Notes

Summary

4m 12s

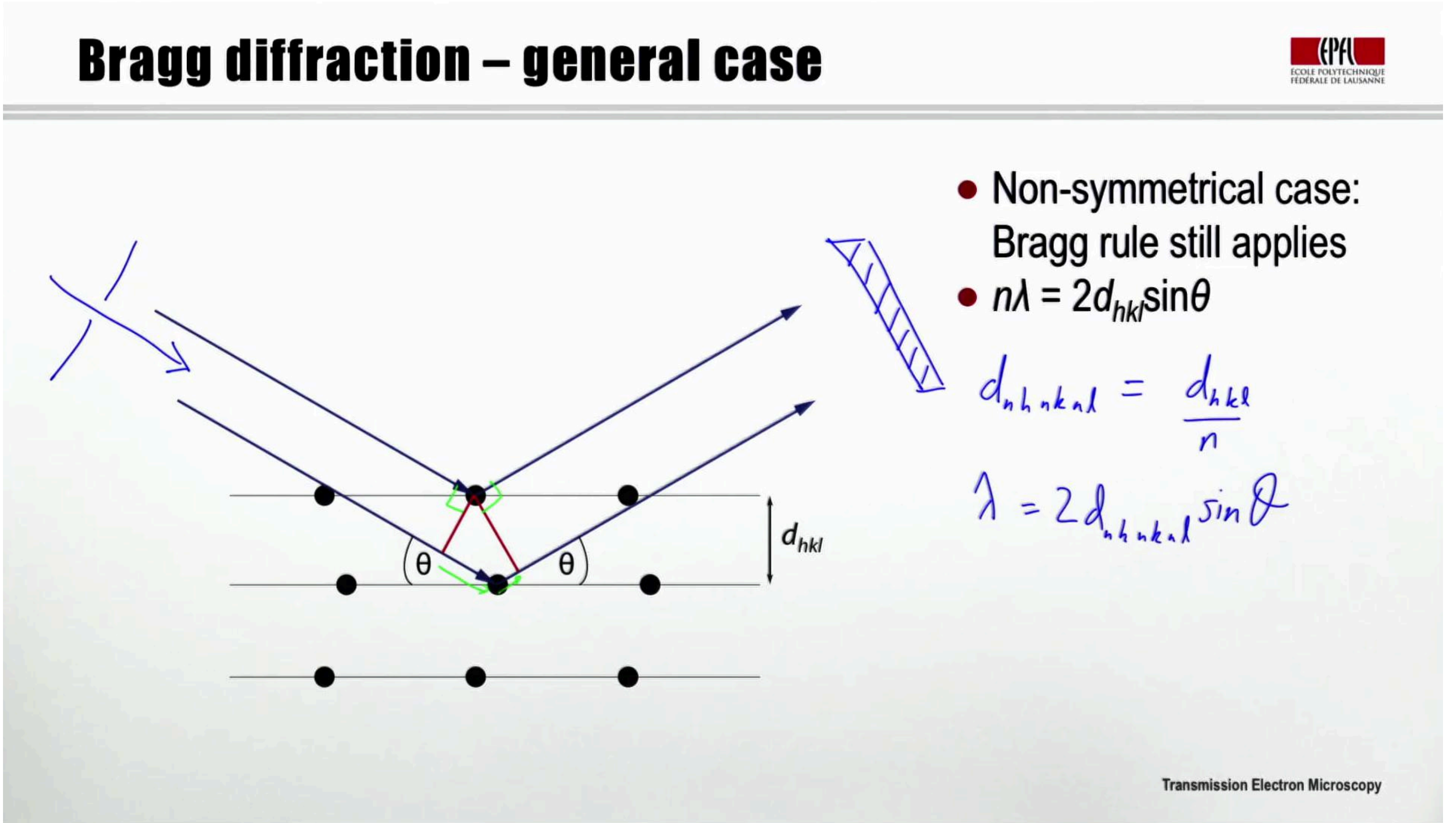



This extra path difference will be the same d sine theta. So, the total extra path traveled by the bottom ray equals 2 d sine theta, where I'm using d as a shorthand for d_hkl. And we know that if this extra path difference 2 d sine theta equals a whole number of wavelengths, we will have constructive interference and hence Bragg diffraction, where here lambda is the wavelength of the electron and n is an integer.

Notes

Summary

6m 02s

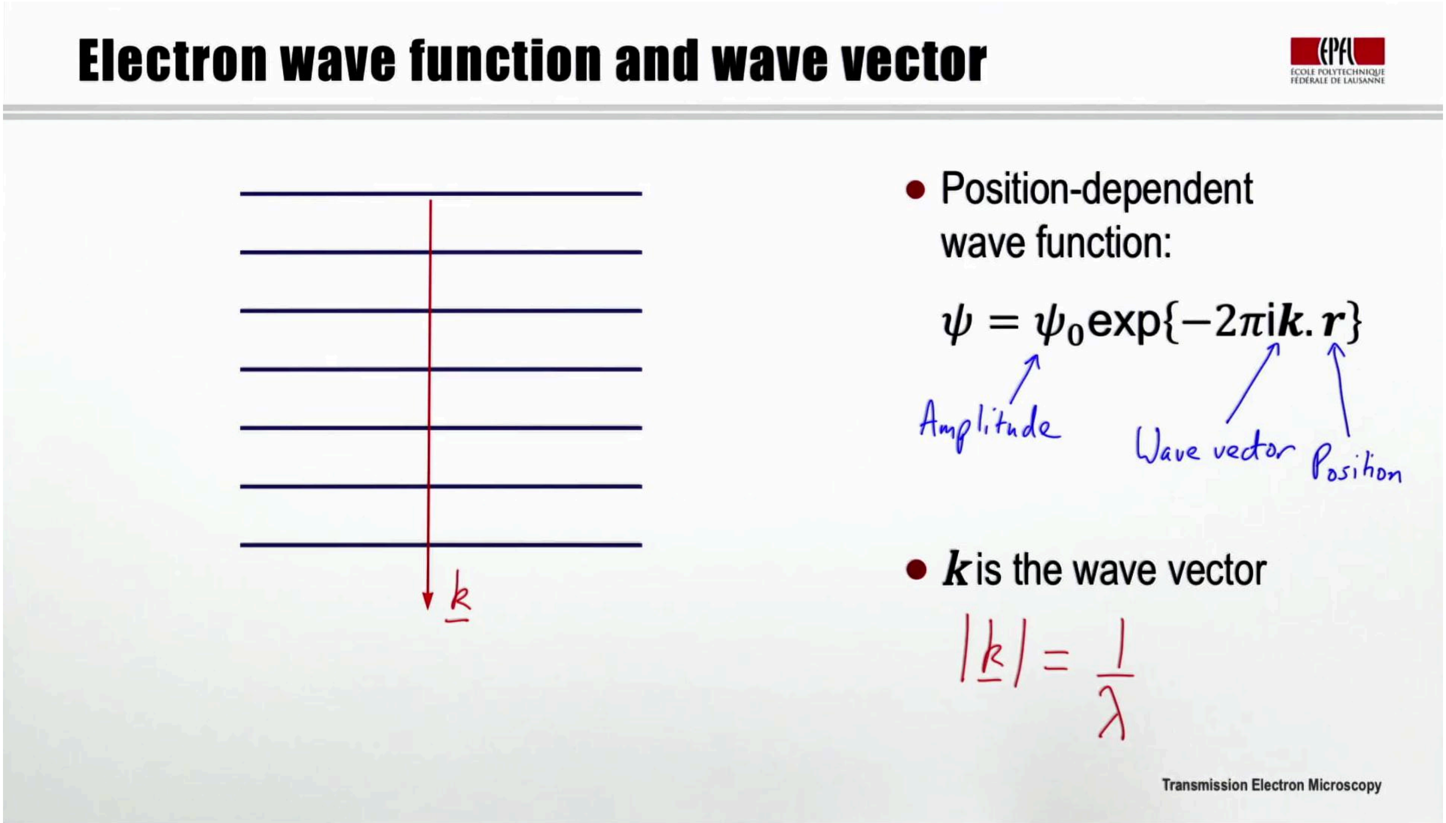



So on the previous slide, we considered Bragg diffraction for a symmetrical case. In a non-symmetrical case, such as here, where the atoms in the lower plane are laterally displaced from those in the upper plane, where the path difference is now marked on as so, we still obtain the same Bragg equation: n lambda = 2 d sine theta. One thing that is helpful with this formula is that we can rewrite it and say that, plane spacing for d_nhnknl equals the plane spacing of d_hkl divided by n. This allows us to rewrite that equation there as: lambda = 2 d_nhnknl sine theta. Doing this allows us to remove the n from the expression in later slides. Now, I said earlier that this diagram here corresponds to the X-ray case. One reason for this is that, in typical X-ray spectrometers, we have the incident beam is produced at an incident angle rather like this, and we have the detector over here. Now in the TEM, instead the incident beam will come down the optic axis from the electron gun. So, in the next slide, we are just going to look at the TEM case, by turning this diagram on its side. But before we do that, I'm going to introduce the wave function and the wave vector, to help us interpret diffraction more accurately.

Notes

Summary

6m 33s

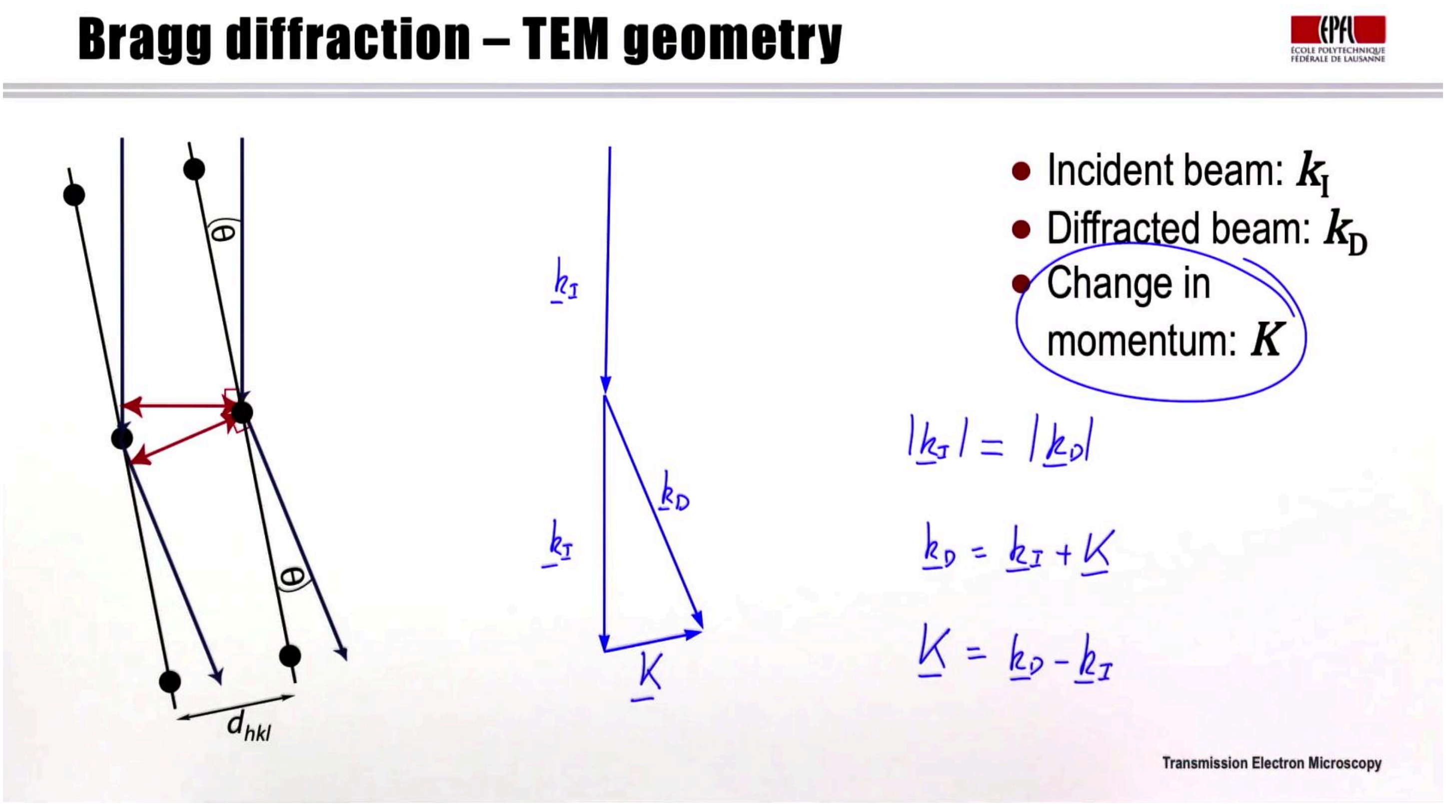



The electron wave can be described by a wave function psi, where: psi = psi_0 times the exponential of –2 pi i k. r where in this expression psi_0 is the amplitude of the wave; k is the wave vector; and r is the position. And between them, 2 pi i k. r gives a phase term in function of position. Here we described just a position dependent wave function, ignoring the time dependency term. Now we are going to look at this wave vector a bit closer. This wave function I have written is for an incident plane parallel wave, as so. And its direction can be described by a wave vector: k. You can mark on this vector here. k is the wave vector and its modulus equals one over lambda, where lambda is the wavelength of the electron.

Notes

Summary

8m 17s

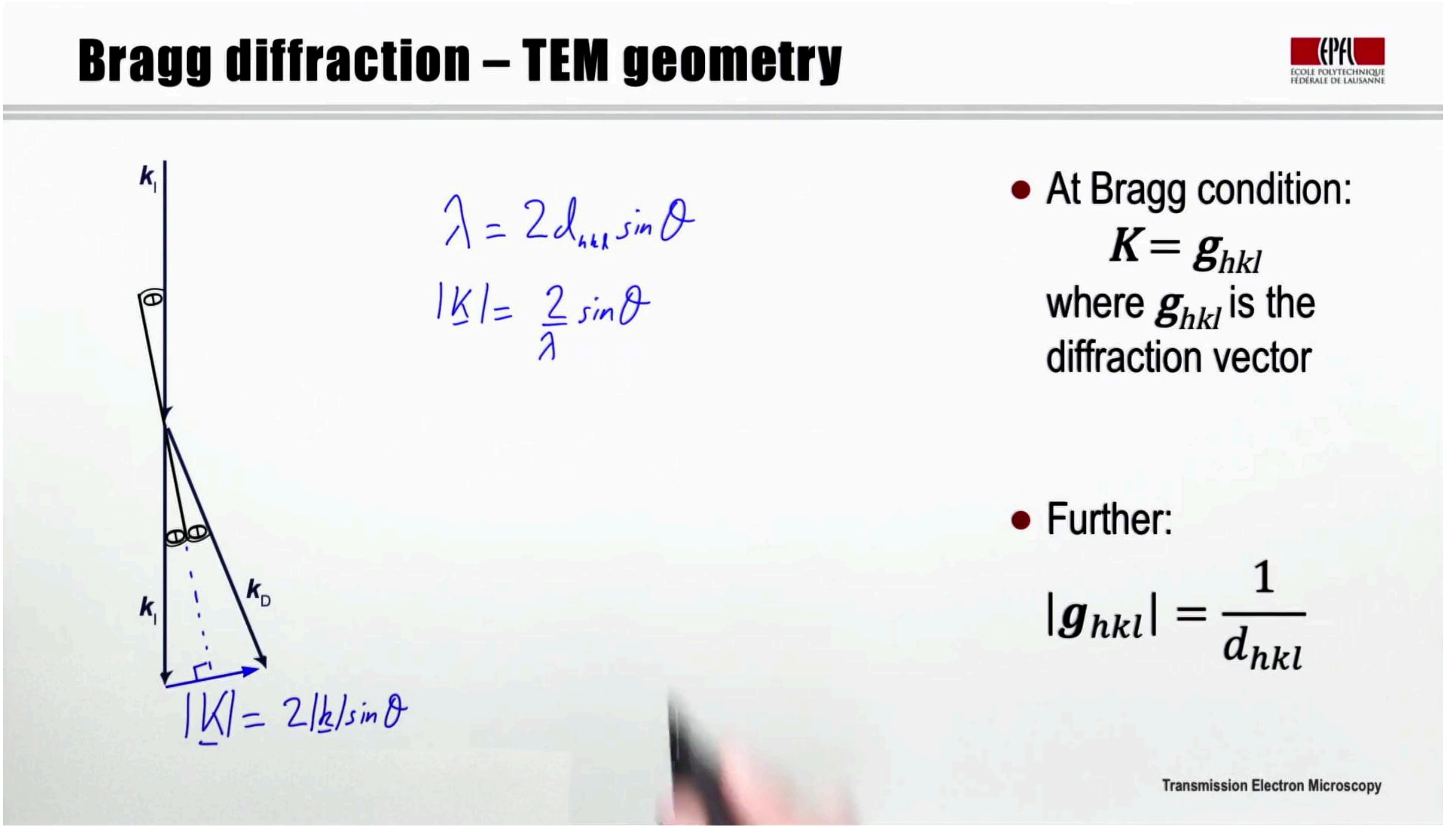



On this slide, I have redrawn the Bragg diffraction schematic for the TEM geometry. So now the incident rays are coming vertically down, as they come down the optic axis of the column. The Bragg diffracting planes are inclined at a small angle theta relative to these incident rays. When there is Bragg diffraction, those rays are scattered at the same angle theta. I am now going to redraw that diagram in terms of the wave vectors for the incident beam, k_I and for the diffracted beam, k_D. So, we start off with the wave vector for the incident electron beam, k_I, and then put in the wave vector for the diffracted electron beam, k_D. Now, because we are considering elastic scattering, there is no change in energy during this scattering event. Because of this, the modulus of k_I, which equals one over lambda, equals the modulus of k_D. On the other hand, there is a change in momentum during the scattering event. So, if first of all we consider the incident wave vector carrying straight on, the difference between these two wave vectors represents that change in momentum. And we can add in an extra vector – here – to represent that change in momentum, which I'm going to call K. We can now see the wave vector for the diffracted beam k_D = k_I + K where that K represents the change in momentum. Equally, we can say that: K = k_D – k_I.

Notes

Summary

9m 19s

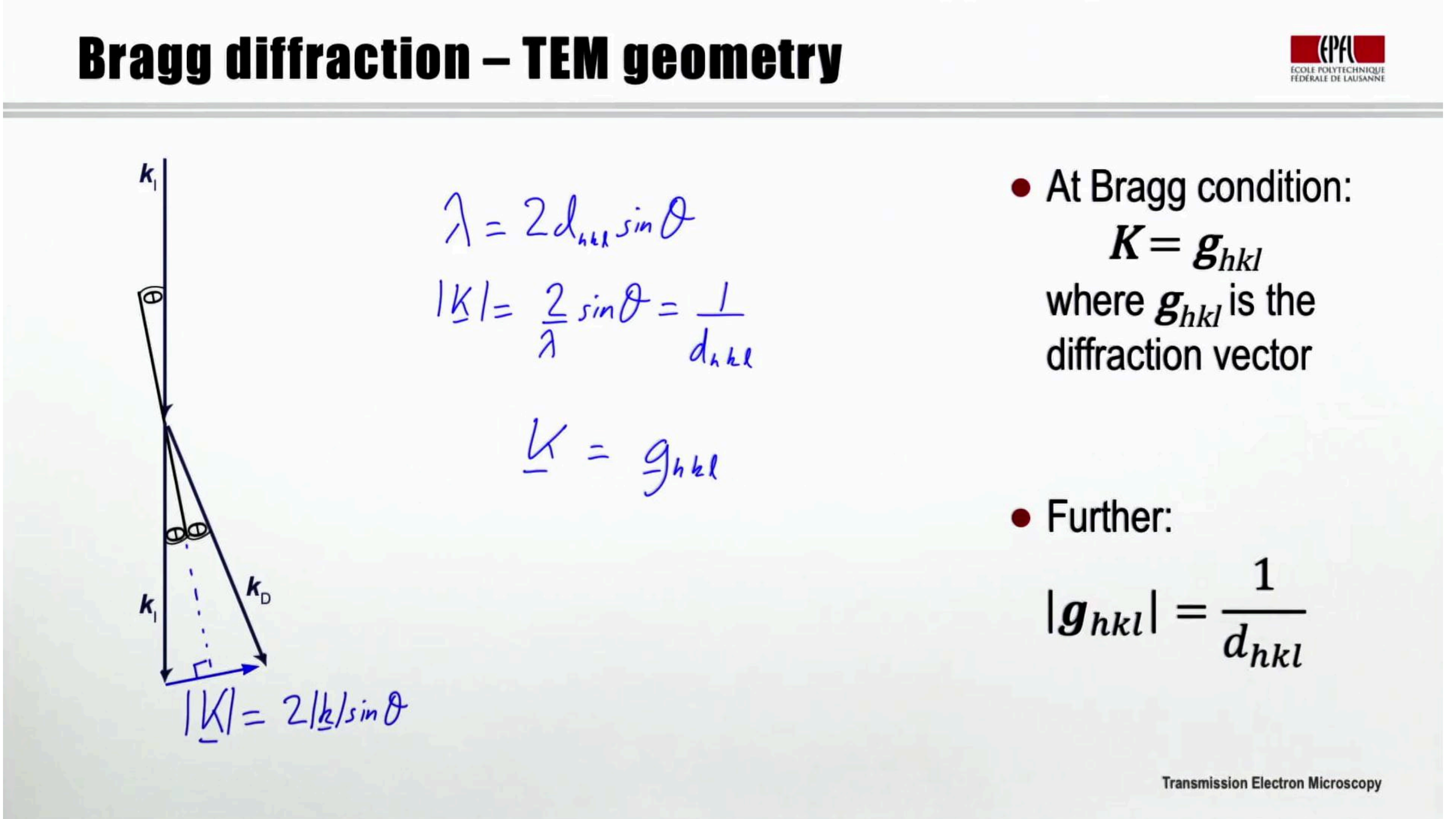



On this slide, I'm going to develop that wave vector diagram a bit further. So, to recap, we have the wave vector k_I for the incident beam, which carries straight on; here. Here we have a plane scattering the incident beam at the Bragg angle theta, generating another beam with the wave vector k_D. And that diffraction can be described by this extra vector that we introduced, which we called K. Now, if we look at the length of that wave vector K – if we draw on a perpendicular here – we can see that half of the length of that vector is clearly going to be the modulus of k times sine theta. So, that means that the modulus of this K overall will equal two times the modulus of k, the general wave vector, times sine theta. And we are going to compare this expression now to the Bragg equation. So, the simplified Bragg equation, which we had developed earlier, was: lambda = 2 d_hkl sine theta. Now, if we look again at this expression for K, the diffraction vector, we have the modulus of K equals two times the modulus of the wave vector – the modulus of the wave vector was one over lambda – so, this is going to be: 2 over lambda times sine theta.

Notes

Summary

11m 29s

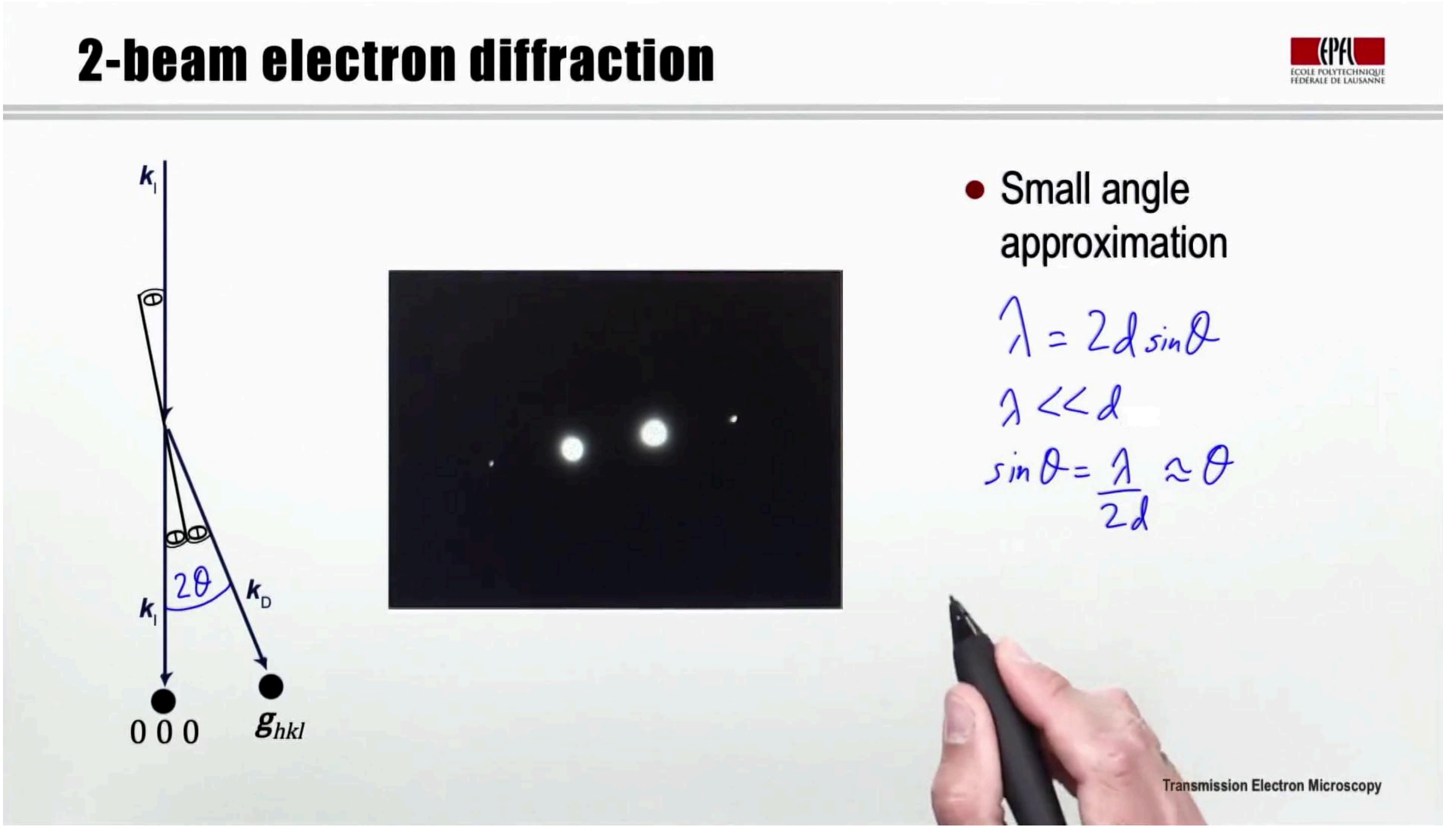



From the Bragg equation we can see that: 2 over lambda sine theta simply equals 1 over d_hkl. So, this equals 1 over d_hkl. In other words, the length of this vector is the reciprocal of the plane spacing that is doing the diffraction. So, we can easily see that Bragg equation in this diagram. Out of this, we are going to describe that vector as the diffraction vector g_hkl. K = g_hkl, where the modulus of that g_hkl equals one divided by the plane spacing. So, we have a reciprocal relationship.

Notes

Summary

13m 21s

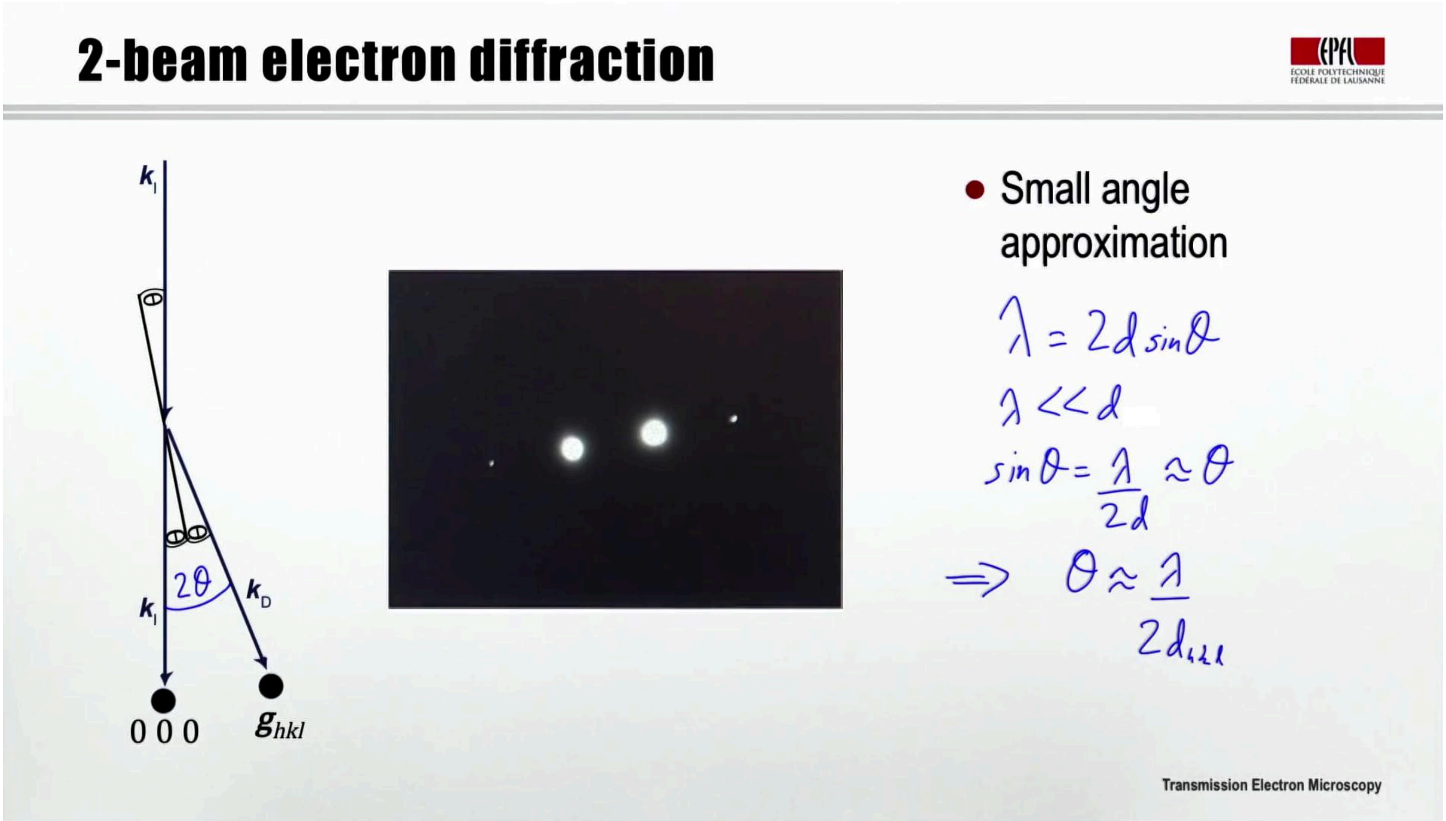



So, how does this Bragg scattering look like in a real experimental case in the electron microscope? So, here is our schematic diagram, where now not only have I marked in the wave vector for the incident beam, the wave vector for the diffracted beam, but also I have symbolically included spots here, for what we would see in the diffraction pattern. So, a spot for the incident beam, which we call 0 0 0. And then another spot, that you see in the diffraction pattern, for the diffracted beam, described by the diffraction vector g_hkl. And the two of them are separated by an angle two theta. Now, in this condition, as we have already said, we have inclined the sample such that there is one plane, (h k l), which is in a strong scattering condition. And when we have done that, indeed we do obtain two bright diffraction spots in the diffraction pattern. When we are in this condition, we can also apply a small angle approximation. From the Bragg equation we know that, lambda = 2 d sine theta. And because the wavelength of the fast electron beam is very small, we know that lambda is much less than d. Since sine theta = lambda over 2 d, and lambda is much smaller than d, this means that sine theta is approximately equal to theta.

Notes

Summary

14m 12s

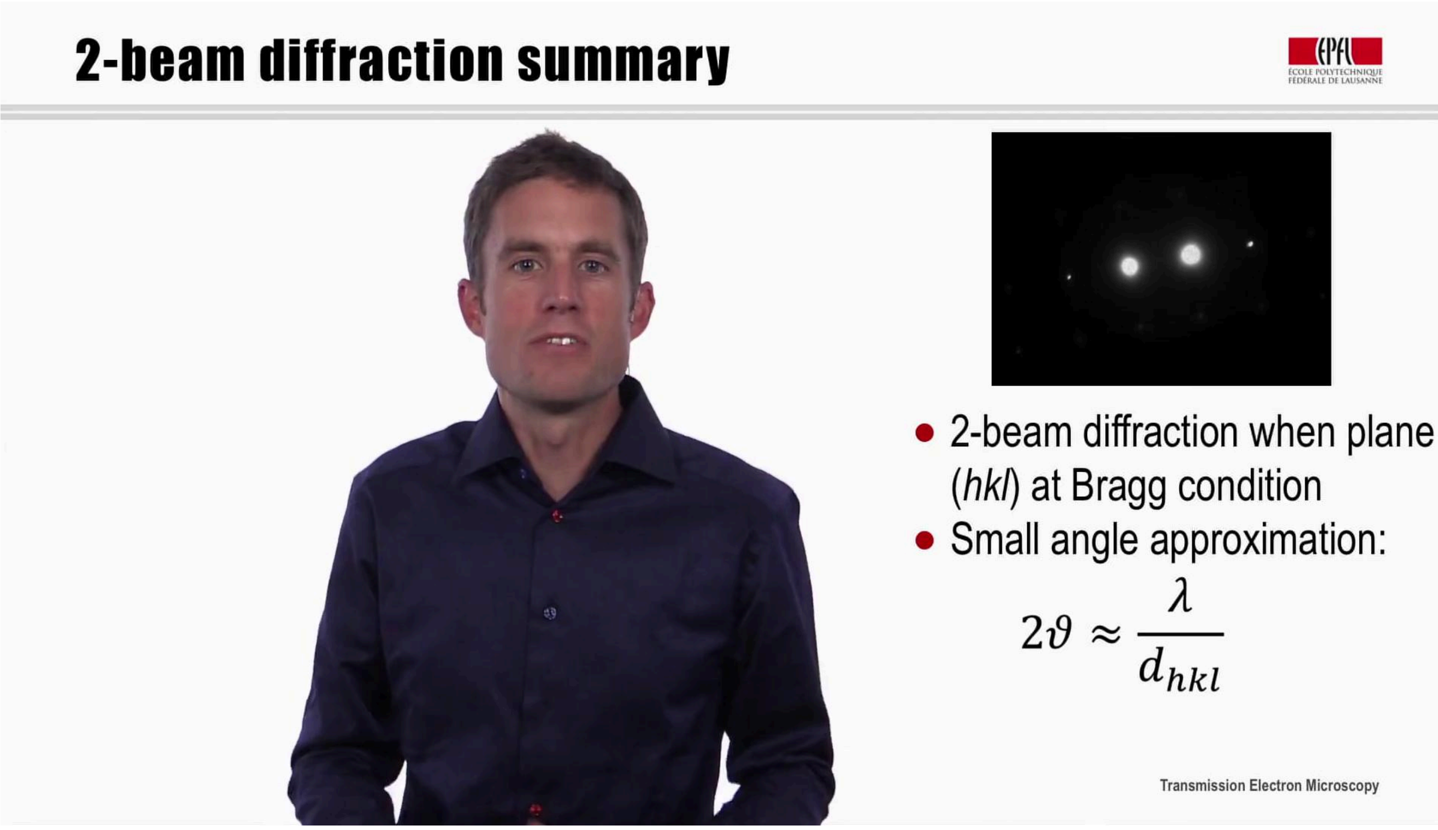



By doing this, we can rewrite the Bragg equation as: theta – our scattering angle – is approximately equal to lambda over 2 d_hkl. What we see in this, is firstly a reciprocal relationship. That, in other words, the smaller the plane spacing, the larger the scattering angle we will obtain. Also, we see that, in this small angle approximation, we have a linearity of scattering angle with the reciprocal of the plane spacing. And this is very useful because it means that we can measure our electron diffraction patterns in a simple linear way, to obtain reciprocal plane spacings.

Notes

Summary

15m 53s




To summarize on this lecture on two-beam electron diffraction, we have seen that, by inclining a crystal in the electron microscope, such that it has one plane (h k l) at the perfect Bragg angle relative to the incident beam, we will have strong scattering from that plane giving a bright diffraction spot in the electron diffraction pattern.

Notes

Summary

16m 45s